Full-duplex MIMO:

Spatial Processing and Characterization

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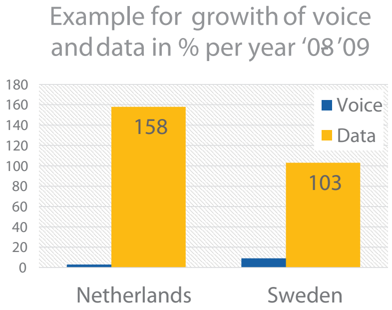
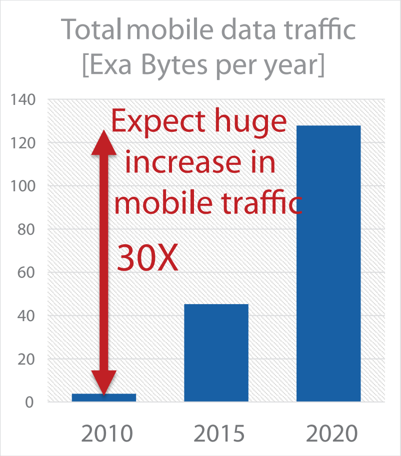
CTW, May 26, 2014

What is ahead of us?... The capacity challenge

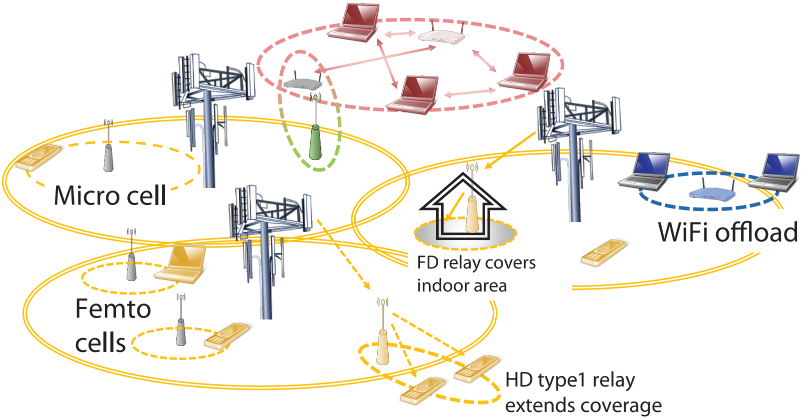
- Number of subscribers saturates at a penetration slightly above 100
- Usage changes: from voice to data





Source: UMTS Forum Report 44 forecasts 2010-2020 report

Future networks will rely on small (femto) cells and WiFi offload



Strong need for flexible *short range links* with high capacity, *flexible spectrum usage*, and for efficient relaying.

Bi-directional wireless communications

Half-duplex (HD) - all wireless communication systems use this

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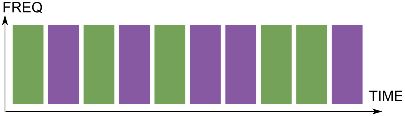
Time-division duplexing (TDD)

Wasted time resources: switching interval



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Frequency-division duplexing (FDD)

Wasted frequency resources: guard bands

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Full-duplex (FD) - a new and efficient alternative

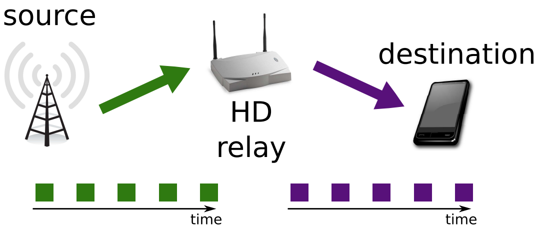
Up to twice the throughput!

No additional transmit power or bandwidth

No wasted time or frequency resources

Improved relaying

HD relay needs to alternate between *reception* and *transmission*



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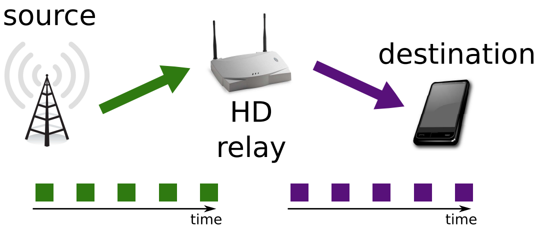
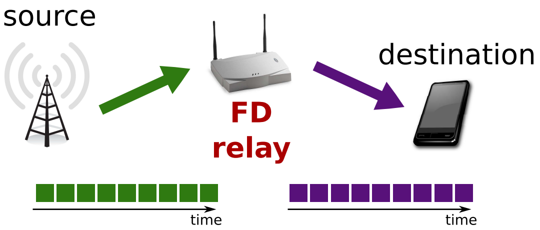


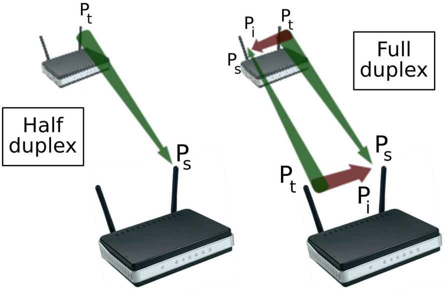
FD relay provides continuous reception and transmission



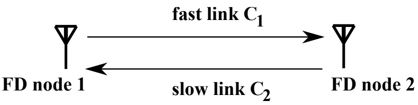
How to compare HD and FD

Transmit power allocation is critical

- Higher transmit power in HD simply improves the quality of the link
- In FD, with higher transmit power we get:
  - improved forward link
  - higher self-interference
- Need to determine the best transmit power to use in FD



A look at the capacities



- Optimization problem:

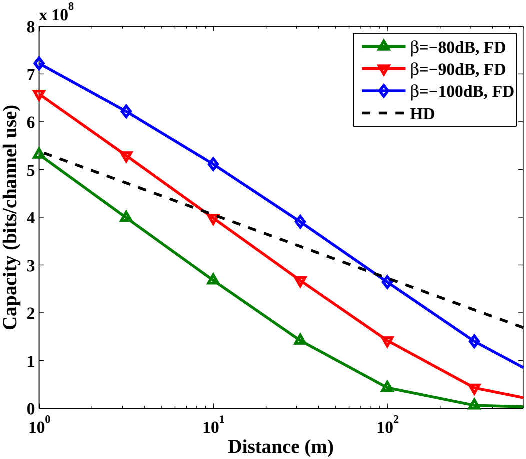
$$\begin{array}{ll} \max_{P_1, P_2} & (1+\alpha)\, C_1 \\ \text{s.t.} & C_2 = \alpha C_1 \\ & P_1 \leq P/2 \\ & P_2 \leq P/2 \end{array}$$

where:

$$C_1 = W \log_2 \left(1 + \frac{\delta P_2}{N_0 + \beta P_1}\right), \; C_2 = W \log_2 \left(1 + \frac{\delta P_1}{N_0 + \beta P_2}\right)$$

W : bandwidth, δ : path loss, β : suppression, P_1, P_2: transmit power

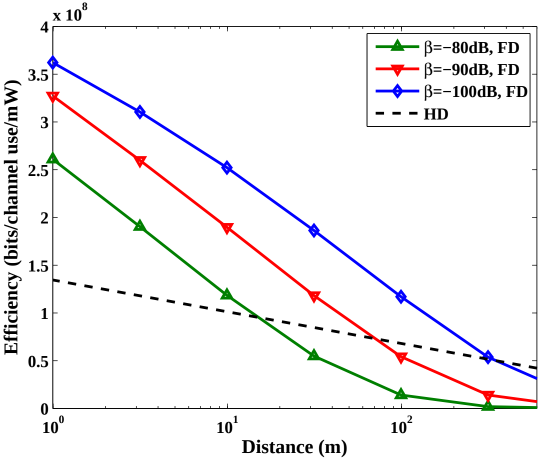
Comparing HD and FD: capacity



FD can provide better capacity than HD!

More self-interference suppression (β) $\Rightarrow$ higher FD gain

Comparing HD and FD: energy efficiency



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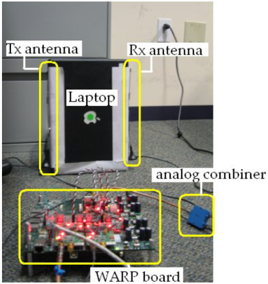
Outline

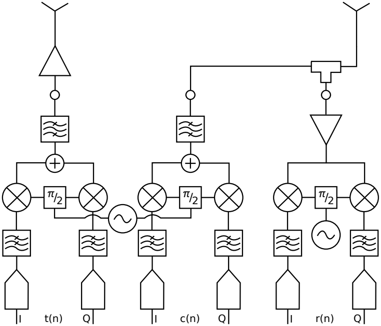
Digital construction of cancelation signal

A flexible (and well-suited for MIMO) way of achieving cancellation

- Cancellation signal constructed in the **digital domain**
- Uses an **additional transmitter**
- First built using WARP boards



(photo: A. Sahai et al./Rice University)



A note on phase noise

- **Phase noise**: limiting factor in FD radios (Sahai 2013, Syrjala 2014)

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Solution: share carrier between Cx and Tx $\rightarrow$ similar phase noise

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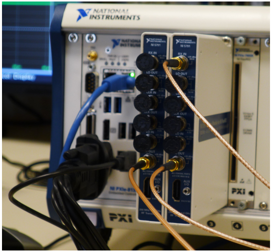

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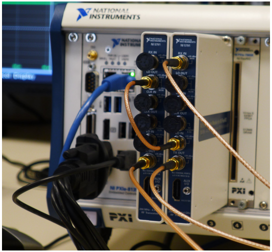

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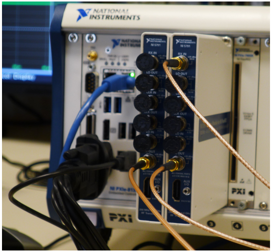

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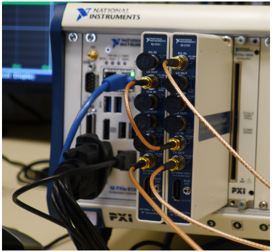

The same approach can reduce the impact of **sampling clock jitter**

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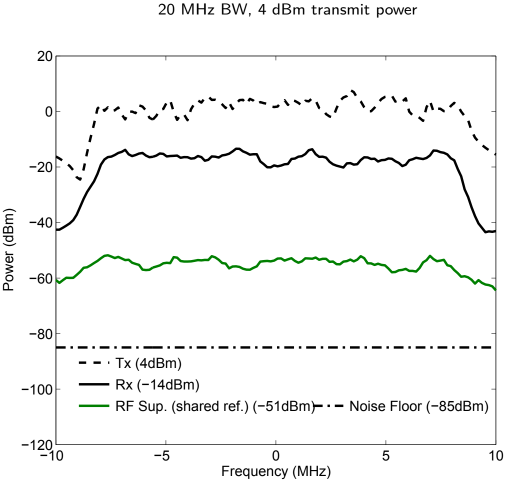
Cancelation results



- Passive analog: **-18 dB**
- Active analog

Linear: **-37 dB**

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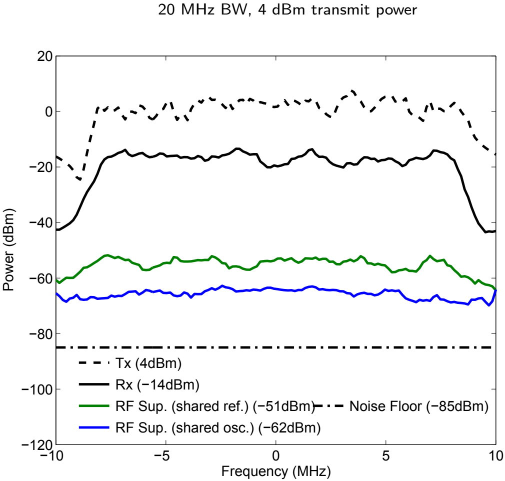

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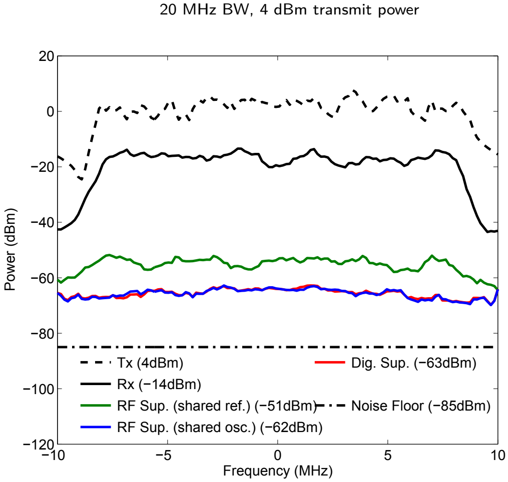

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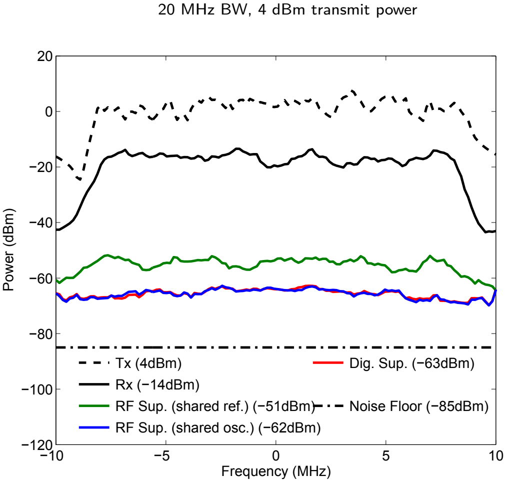

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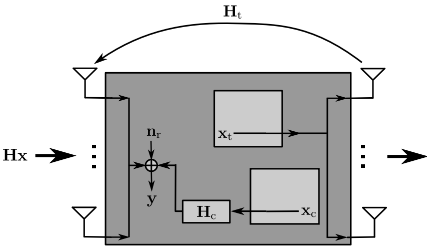
- Total: **-67 dB**

- Residual power: **-63 dBm**

Full-Duplex MIMO

- No cancellation:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{H}_t\mathbf{x}_t + \mathbf{n}_r$$



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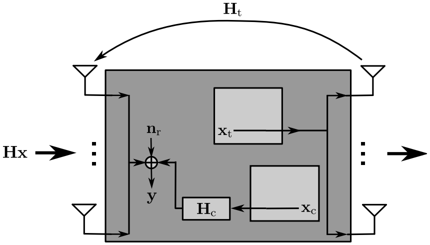

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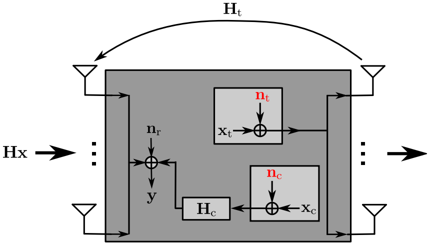

- In practice, transmitted signals are affected by **non-idealities**:

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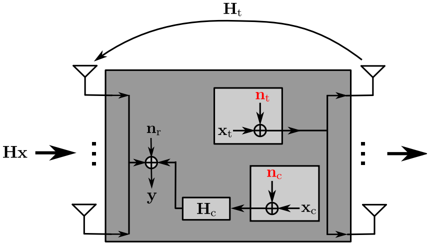

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- Cancellation under transmit impairments:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{H}_\mathsf{t}\tilde{\mathbf{x}}_\mathsf{t} + \mathbf{H}_\mathsf{c}\tilde{\mathbf{x}}_\mathsf{c} + \mathbf{n}_\mathsf{r} = \mathbf{H}\mathbf{x} + \mathbf{H}_\mathsf{t}\mathbf{n}_\mathsf{t} + \mathbf{H}_\mathsf{c}\mathbf{n}_\mathsf{c} + \mathbf{n}_\mathsf{r}$$

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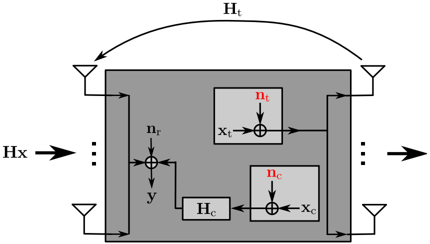

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Effective noise: $\mathbf{n}_{\text{eff}} \triangleq \mathbf{H}_t\mathbf{n}_t + \mathbf{H}_c\mathbf{n}_c + \mathbf{n}_r$

2×2 Full-Duplex MIMO Testbed hardware

- We wish to characterize the effective noise $\mathbf{n}_{\text{eff}}$ to:
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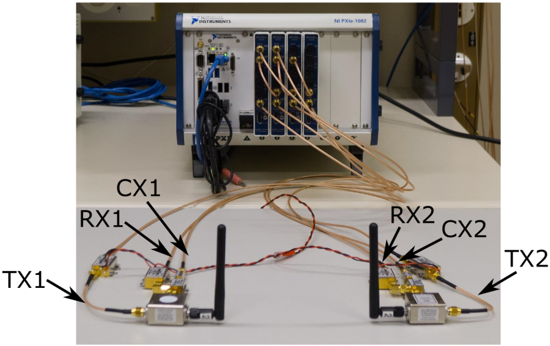

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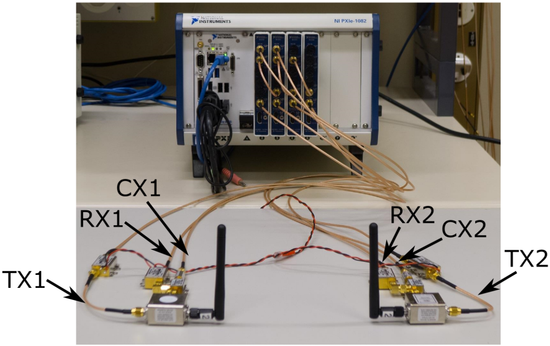
  - Runs Windows with LabVIEW



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  3. Histograms (to assess **distribution**)
  4. Spatial covariance matrix (to assess **spatial correlation**)

Autocorrelation

- The autocorrelation of each element of $\mathbf{n}_{\text{eff}}$ is estimated as

$$\hat{\mathbf{R}}_{i,j} = \begin{cases} \sum_{k=0}^{N-j-1} \mathbf{N}_{i,j+k} \mathbf{N}_{i,k}^*, & j \geq 0, \\ \hat{\mathbf{R}}_{i,-j}^*, & j < 0, \end{cases}$$

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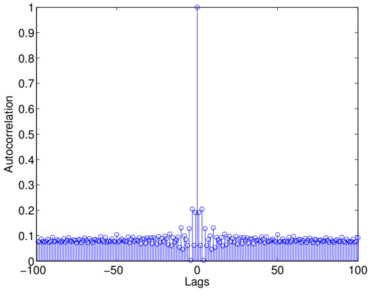

Time domain

- **Time domain:** $\mathbf{n}_{\text{eff}}$ has **non-negligible memory**

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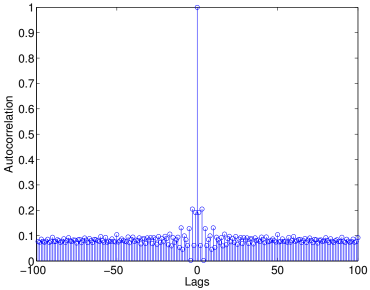

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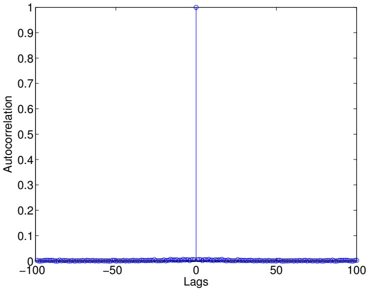

Frequency domain

- **Time domain:** $\mathbf{n}_{\text{eff}}$ has **non-negligible memory**
- **Frequency domain:** $\mathbf{n}_{\text{eff}}$ is practically **memoryless**

Pseudo-variance

- For each chain i, the pseudo-variance is defined as:

$$\tau_i^2 \triangleq \mathbb{E}\left[\mathbf{n}_{\text{eff},i}^2\right]$$

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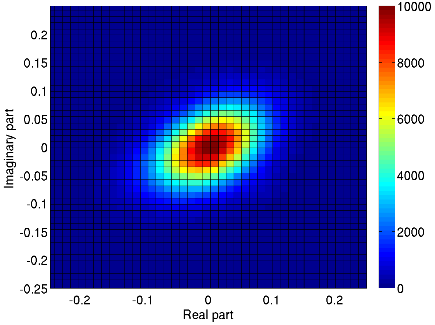
- **Time domain:** $|\hat{\tau}_1^2| \approx 10^{-3}$
- **Frequency domain:** $|\hat{\tau}_1^2| \approx 10^{-5} \rightarrow$ **more circular**

Histograms

- Joint histogram of $\Re(\mathbf{N}_{1,j})$ and $\Im(\mathbf{N}_{1,j})$



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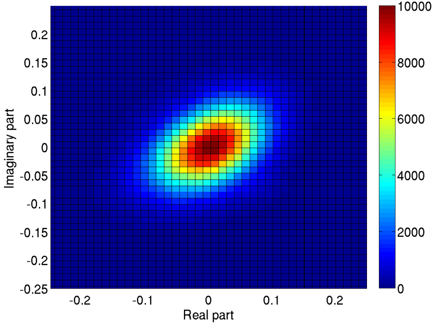

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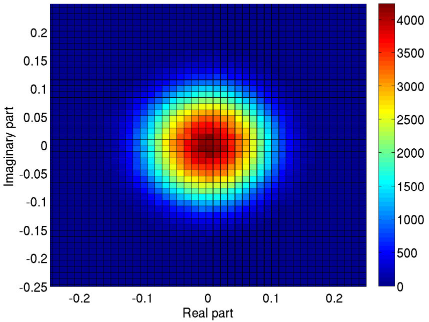

Frequency domain

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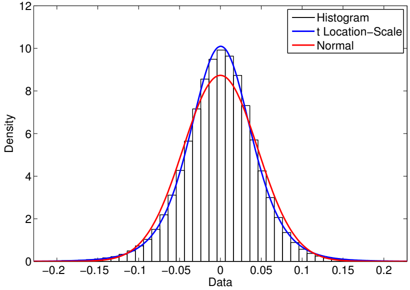

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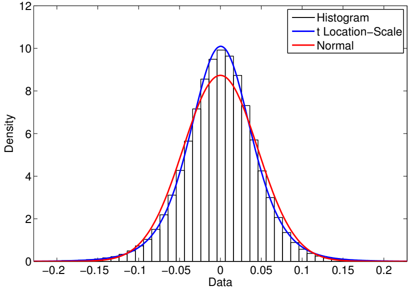

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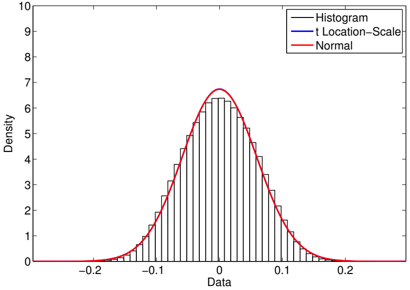

Frequency domain

- **Time domain: Not Gaussian** (Student's t-distribution is good fit)
- **Frequency domain: Gaussian** (central limit theorem)

Spatial covariance matrices

- Spatial covariance matrix: $\mathbf{K} \triangleq \mathbb{E}\left[\left(\mathbf{n}_{\text{eff}} - \mathbb{E}[\mathbf{n}_{\text{eff}}]\right)\left(\mathbf{n}_{\text{eff}} - \mathbb{E}[\mathbf{n}_{\text{eff}}]\right)^H\right]$

Measurements are specific to our setup. However, the variance of $\hat{\mathbf{K}}_{\text{time}}$ over time and $\hat{\mathbf{K}}_{\text{freq}}$ over the frequency tones is small compared to the magnitude of the entries.

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$$\hat{\mathbf{K}}_{\text{time}} = \begin{bmatrix} 0.0067 & -0.0013 - 0.0031i \\ -0.0013 + 0.0031i & 0.0053 \end{bmatrix}$$

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OFDM: Need to **study** and **undo** effects of colored noise

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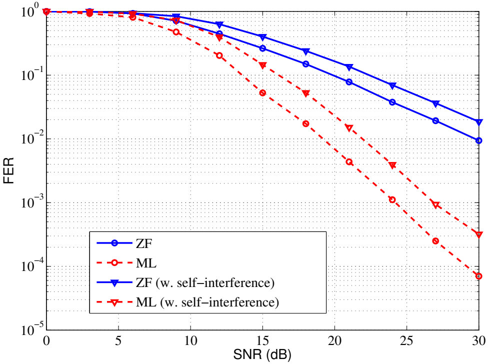

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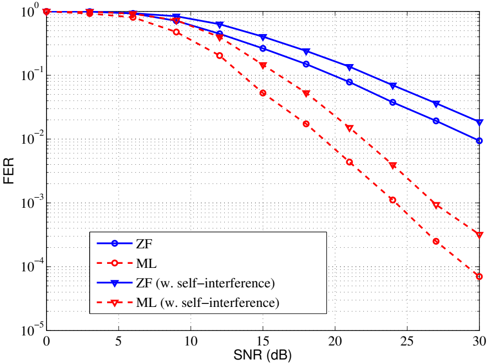

Colored noise → **∼3 dB** worse performance

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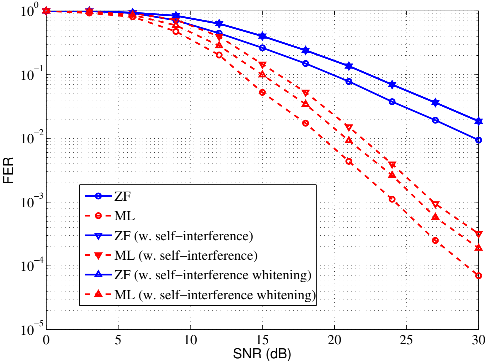

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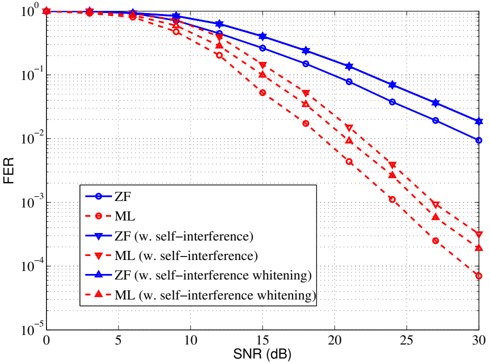

ML: Noise whitening → ∼**1 dB** reclaimed

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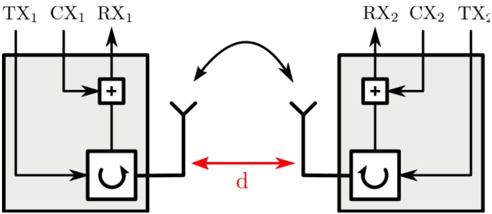
- $\mathbf{K}$ can be estimated in training phase
  - We have observed that $\mathbf{K}$ does not vary significantly with low mobility
- Since the setup is highly static, we can attempt to build a model to predict $\mathbf{K}$
  - No need to estimate $\mathbf{K}$
  - **Possibility of optimizing the setup to reduce coloring**

Setup



- Two RF chains, **antenna distance d**

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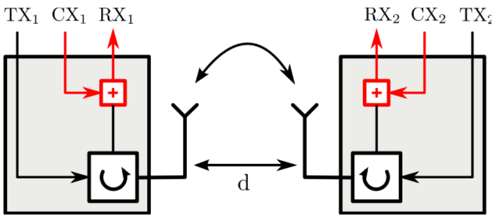

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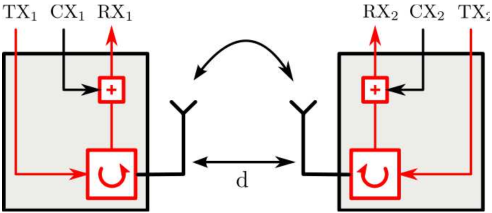

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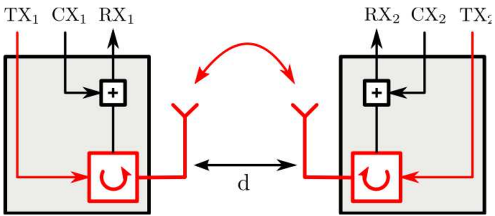

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- Self-interference channel from transmitter i to receiver i is constant $\rightarrow$ modeled as **constant gain** β and **constant phase** ϕ_β:

$$\mathbf{H}_\mathsf{t} = \begin{bmatrix} \beta\, e^{j\phi_\beta} & ? \\ ? & \beta\, e^{j\phi_\beta} \end{bmatrix}$$

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- Then, we get

$$\mathbf{K}_{\mathbf{y}}(d) = \begin{bmatrix} A(d) & B(d) \\ B(d) & A(d) \end{bmatrix},$$

where

$$A(d) = \alpha^2 + \beta^2 + \gamma(d)^2 + \sigma^2$$

and

$$B(d) = \beta\gamma(d)\left(e^{j(\phi_\gamma(d)-\phi_\beta)} + e^{-j(\phi_\gamma(d)-\phi_\beta)}\right)$$

Avoiding colored noise

- **Optimal distance** d^* to minimize off-diagonal elements (i.e., minimize spatial correlation):

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$$d^* = \left(\frac{2k+1}{4} - \frac{\phi_\beta}{2\pi} \right) \lambda, \quad k \in \mathbb{Z},$$

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- Suitably chosen antenna spacing **eliminates coloring**

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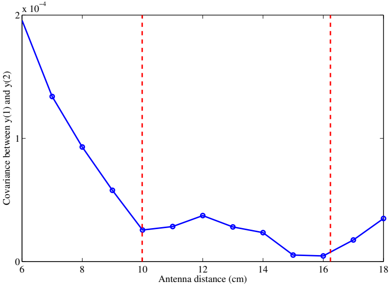

Measurements

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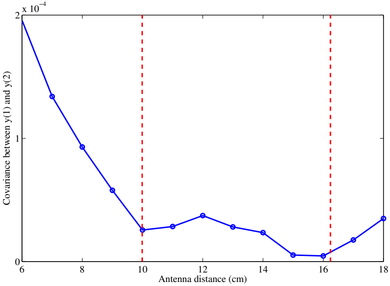
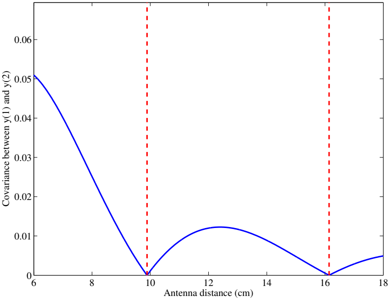


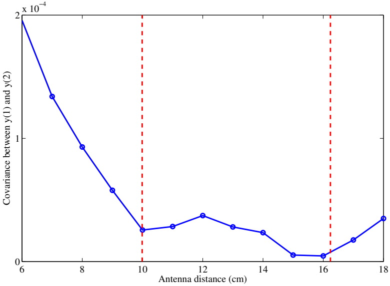
Measurements



Model

Covariance matrix model verification

- Carrier frequency: **2.40 GHz**, signal bandwidth: **10 KHz**



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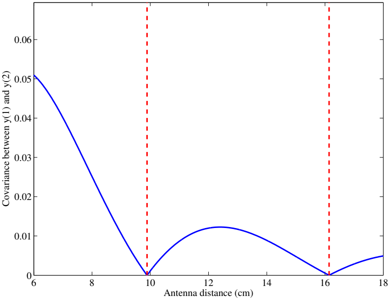

Model

- Initial measurements indicate **good agreement**

Conclusions

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- Due to **static nature** of the setup correlation can be captured by a simple geometric model
- Antenna position can be **optimized** to reduce correlation (for narrowband signals)